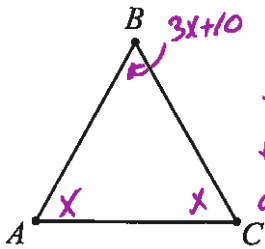


## Triangle Properties - Triangle Angle Sum Theorem & Exterior Angle Theorem

**Triangle Angle Sum Theorem:** The angles of a triangle sum to 180 degrees.



**Example:** Given  $\angle A \cong \angle C$  and  $m\angle B$  is 10 more than three times  $m\angle A$ . Find  $m\angle B$ .

Let  $m\angle A = x$ .  
then  $m\angle C = x$   
and  $m\angle B = 3x + 10$

$$x + x + 3x + 10 = 180$$

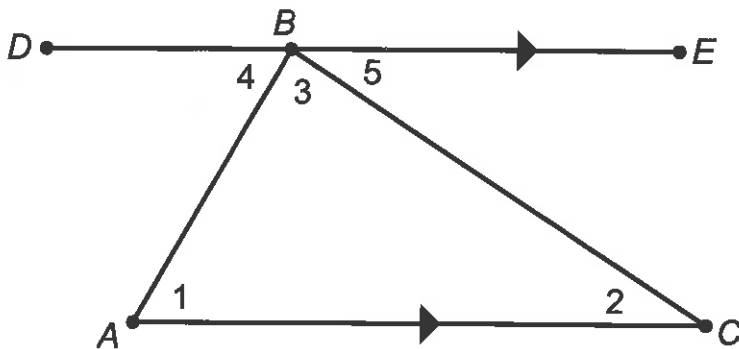
$$5x + 10 = 180$$

$$5x = 170$$

$$x = 34$$

$$m\angle B = 3(34) + 10 = 112^\circ$$

How can ~~this~~ the Triangle Sum Theorem be proven true?



To prove the theorem true, we first draw a line parallel to one side of the triangle as shown in the diagram. Using the parallel lines we can justify that  $m\angle 1 + m\angle 2 + m\angle 3 = 180$ .

a. Explain why  $\angle 1 \cong \angle 4$  and  $\angle 2 \cong \angle 5$ .

Because when 2 || lines are cut by a trans. the alt. int.  $\angle$ 's are  $\cong$ .

b. Explain why  $m\angle 4 + m\angle 3 + m\angle 5 = 180$ .

$\angle DBE$  is a straight angle thus  $= 180^\circ$

By angle addition,  $m\angle 4 + m\angle 3 + m\angle 5 = m\angle DBE$

c. Based on your answers from part a and b, explain why  $m\angle 1 + m\angle 2 + m\angle 3 = 180$ .

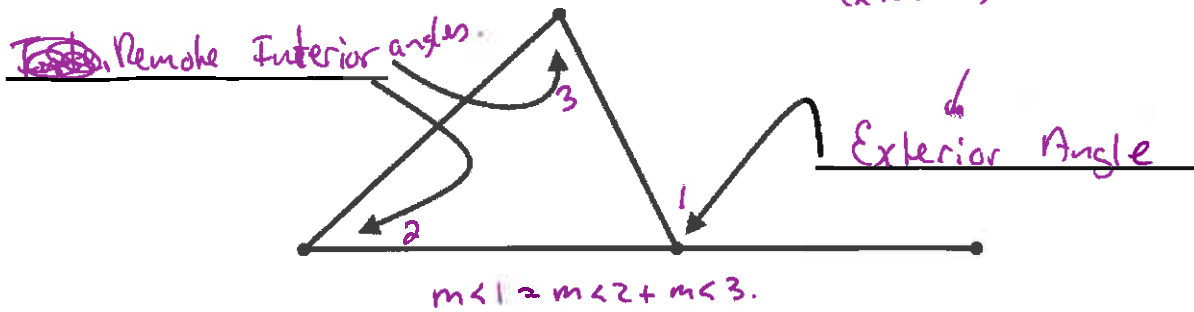
By substituting in  $\angle 1$  for  $\angle 4$  and  $\angle 2$  for  $\angle 5$

into  $m\angle 4 + m\angle 3 + m\angle 5 = 180$ , we get the

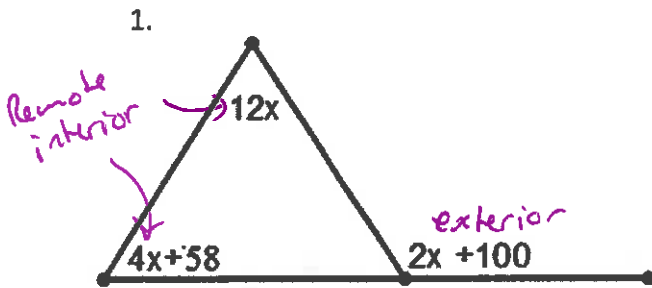
desired.  $m\angle 1 + m\angle 3 + m\angle 2 = 180^\circ$ .

**Exterior Angle Theorem:** An exterior angle of a triangle is equal to the sum of the 2 remote interior angles of the triangle.

*the Exterior  $\angle$  is formed by extending the side of the  $\Delta$ .*



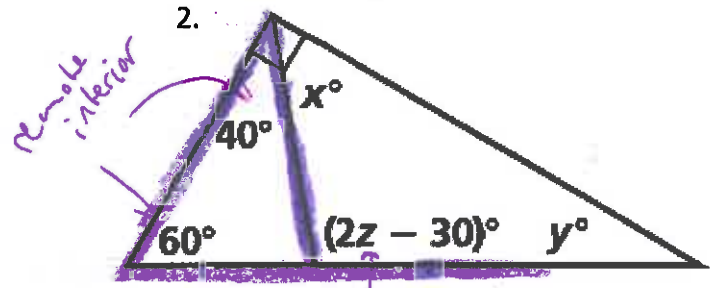
**Example:** Find the value of the variables.



$$2x + 100 = 12x + 4x + 58$$

$$42 = 14x$$

$$x = 3$$



$$2z - 30 = 40 + 60$$

$$2z = 130$$

$$z = 65$$

*Exterior  $\angle$ .*

$$x + 40 = 90$$

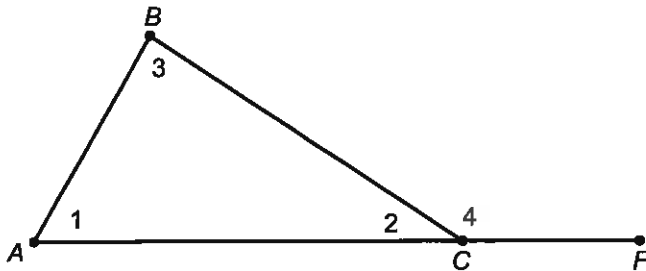
$$x = 50$$

$$x + y + 65 = 180$$

$$50 + y + 65 = 180$$

$$y = 65$$

**How can the Exterior Angle Theorem be proven true?**



To prove the theorem true, we can make use of the Triangle Angle Sum Theorem to justify that  $m\angle 4 = m\angle 1 + m\angle 3$ .

a. Explain why  $m\angle 4 + m\angle 2 = 180$  and  $m\angle 1 + m\angle 2 + m\angle 3 = 180$ .

*Because  $\angle 4$  and  $\angle 2$  are Supplementary.*

*Because  $\angle 1, \angle 2,$  and  $\angle 3$  are Angles of a  $\Delta$ .*

b. Explain why your results from part a make  $m\angle 4 + m\angle 2 = m\angle 1 + m\angle 2 + m\angle 3$

*By substitution (or transitive also works)*

c. Explain why part b makes  $m\angle 4 = m\angle 1 + m\angle 3$

*By subtracting the measure of  $\angle 2$  from both sides of the  $=$ , you get  $m\angle 4 = m\angle 1 + m\angle 3$ .*